LOW-COMPLEXITY SERIAL EQUALIZATION OF DOUBLY-SELECTIVE CHANNELS *

Geert Leus, Imad Barhumi, and Marc Moonen

K.U.Leuven - ESAT
Kasteelpark Arenberg 10, 3001 Leuven, Belgium
Email: {geert.leus,imad.barhumi,marc.moonen}@esat.kuleuven.ac.be

ABSTRACT

In this paper, we propose a serial equalizer for doubly-selective channels. We use a Basis Expansion Model (BEM) Finite Impulse Response (FIR) to model the doubly-selective channel and to design the serial equalizer. In contrast to our previous works, we decrease the modeling error of the BEM FIR by fitting the BEM FIR to the true doubly-selective channel over a time-window that is independent of the frequency-resolution of the BEM FIR. We discuss the Serial Linear Equalizer (SLE) as well as the Serial Decision Feedback Equalizer (SDFE).

1. INTRODUCTION

The need for high data rates and high mobility in future wireless communication systems introduces doubly-selective (time- and frequency-selective) channel effects. To combat these effects, equalizers are needed. In this paper, we propose a serial equalization approach for doubly-selective channels. We use a Basis Expansion Model (BEM) Finite Impulse Response (FIR) to model the doubly-selective channel and to design the serial equalizer. This allows us to turn a high-complexity Time-Varying (TV) 1-dimensional deconvolution problem into an equivalent low-complexity Time-InVariant (TIV) 2-dimensional deconvolution problem, containing only the BEM FIR parameters of both the doubly-selective channel and the serial equalizer. In the past a TIV FIR serial equalizer has been employed to equalize a BEM FIR channel, but this requires many (symbol-rate sampled) receive antennas for the linear zero-forcing solution to exist [11]. However, when a BEM FIR serial equalizer is used to equalize a BEM FIR channel, only two (symbol-rate sampled) receive antennas are required for the linear zero-forcing solution to exist [9, 2].

In contrast to our previous works [9, 2, 1], where the modeling error of the BEM FIR causes a BER performance saturation at high SNR, we introduce a novel approach that eliminates this BER floor. The basic idea is that we fit the BEM FIR to the true doubly-selective channel over a time-window that is independent of the frequency-resolution of the BEM FIR. This allows for an almost perfect fit of the BEM FIR to the true doubly-selective channel, without losing the interesting features of the BEM FIR.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We use $\star$ to denote the convolution operation. We denote the Dirac delta by $\delta(t)$ and the Kronecker delta by $\delta[n]$. We write the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$. Finally, diag$\{\mathbf{x}\}$ represents the diagonal matrix with $\mathbf{x}$ on the diagonal.

* This work was carried out at the ESAT laboratory of the Katholieke Universiteit Leuven, in the frame of the Belgian State's Interuniversity Poles of Attraction Programme (2002-2007) - IUAP P5/22 ('Dynamical Systems and Control: Computation, Identification and Modelling') and P5/11 ('Mobile multimedia communication systems and networks'), the Concerted Research Action GOA-MEFISTO-666 (Mathematical Engineering for Information and Communication Systems Technology) of the Flemish Government, and Research Project FWO nr.G.0196.02 ('Design of efficient communication techniques for wireless time-dispersive multi-user MIMO systems'). Geert Leus is a Postdoctoral Fellow of the FWO - Vlaanderen (Belgium). Imad Barhumi is partly supported by the UNESCO/PEACE Program.

2. CHANNEL MODEL

We consider a baseband description of a wireless system with 1 transmit and M receive antennas. For the mth receive antenna, the symbol sequence $x[n]$ is filtered by the transmit filter $g_{\text{tr}}(t)$, distorted by the physical channel $g_{\text{ch}}^{(m)}(t;\tau)$, corrupted by the additive noise $v^{(m)}(t)$, and finally filtered by the receive filter $g_{\text{rec}}(t)$. The received signal at the mth receive antenna $y^{(m)}(t)$ can then be written as

$$y^{(m)}(t) = \sum_{n=-\infty}^{\infty} g^{(m)}(t;t-nT)x[n] + w^{(m)}(t),$$

where T is the symbol period, $w^{(m)}(t) := g_{\text{rec}}(t) \star v^{(m)}(t)$, and [5, ch. 1]

$$g^{(m)}(t;\tau) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_{\text{rec}}(s)g_{\text{tr}}(\tau-\theta-s)g_{\text{ch}}^{(m)}(t-s;\theta)dsd\theta. \tag{1}$$

Sampling each receive antenna at symbol rate, the received sequence at the mth receive antenna $y^{(m)}[n] := y^{(m)}(nT)$ can be written as

$$y^{(m)}[n] := \sum_{\nu=-\infty}^{\infty} g^{(m)}[n;\nu]x[n-\nu] + w^{(m)}[n], \tag{2}$$

where $w^{(m)}[n] := w^{(m)}(nT)$ and $g^{(m)}[n;\nu] := g^{(m)}(nT;\nu T)$.

Most wireless links experience multipath propagation, where clusters of reflected or scattered rays arrive at the receiver. All the rays within the same cluster experience the same delay, but each of them is characterized by its own complex gain and frequency offset. Hence, we can express the physical channel $g_{\text{ch}}^{(m)}(t;\tau)$ as [8, ch. 1], [3, ch. 3], [4], [5, ch. 1]

$$g_{\text{ch}}^{(m)}(t;\tau) = \sum_c \delta(\tau-\tau_c^{(m)})\sum_r G_{c,r}^{(m)} e^{j2\pi f_{c,r}^{(m)}t}, \tag{3}$$

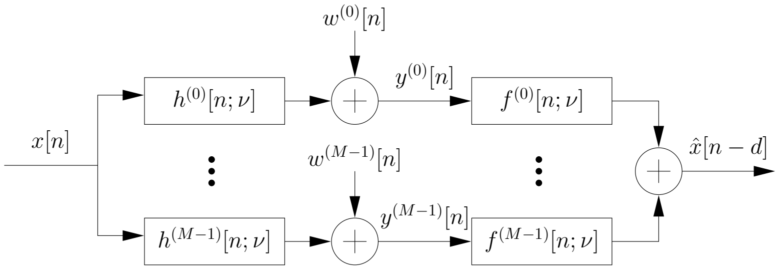

Fig. 1. Serial linear equalization.

where $\tau_c^{(m)}$ is the delay of the cth cluster related to the mth receive antenna, and $G_{c,r}^{(m)}$ and $f_{c,r}^{(m)}$ are respectively the complex gain and frequency offset of the rth ray of the cth cluster related to the mth receive antenna.

Assuming the time-variation of the physical channel $g_{\text{ch}}^{(m)}(t;\tau)$ over the span of the receive filter $g_{\text{rec}}(t)$ is negligible, we can replace $g_{\text{ch}}^{(m)}(t-s;\theta)$ by $g_{\text{ch}}^{(m)}(t;\theta)$ in (1), leading to

$$
\begin{aligned}
g^{(m)}(t;\tau) &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} g_{\text{rec}}(s)g_{\text{tr}}(\tau-\theta-s)ds\right) g_{\text{ch}}^{(m)}(t;\theta)d\theta \\
&= \int_{-\infty}^{\infty}\psi(\tau-\theta)g_{\text{ch}}^{(m)}(t;\theta)d\theta \\
&= \sum_c \psi(\tau-\tau_c^{(m)})\sum_r G_{c,r}^{(m)}e^{j2\pi f_{c,r}^{(m)}t},
\end{aligned}
$$

where $\psi(t) := g_{\text{rec}}(t) \star g_{\text{tr}}(t)$. This means that $g^{(m)}[n;\nu]$ can be expressed as

$$
\begin{aligned}
g^{(m)}[n;\nu] &= g^{(m)}(nT;\nu T) \\
&= \sum_c \psi(\nu T-\tau_c^{(m)})\sum_r G_{c,r}^{(m)}e^{j2\pi f_{c,r}^{(m)}nT}. \qquad (4)
\end{aligned}
$$

The above channel model has a rather complex structure, which complicates, if not prevents, the development of a low-complexity equalization structure that blends well with the channel structure. Moreover, the above channel model contains a large number of parameters, which causes a major problem when trying to estimate the channel. Hence, we try to develop a channel model that is well-structured and contains a small number of parameters. Therefore, we will look at a limited time window $t \in [0, NT)$, which corresponds to $n \in \{0,1,\ldots,N-1\}$. Assuming $g^{(m)}(t;\tau) = 0$ for $\tau \notin [0,(L+1)T)$, each channel $g^{(m)}[n;\nu]$ can be modeled for $n \in \{0,1,\ldots,N-1\}$ by a so-called BEM FIR:

$$
h^{(m)}[n;\nu] = \sum_{l=0}^{L}\delta[\nu-l]\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(m)}e^{j2\pi qn/K}, \qquad (5)
$$

where Q and K should be selected such that $Q/(2KT) \approx f_{\max}$, with $f_{\max}$ the overall Doppler spread of all M channels:

$$
f_{\max} := \max_{m,c,r}\{|f_{c,r}^{(m)}|\}.
$$

The big difference between this BEM FIR and the BEM FIR that we used in our previous works [9, 2, 1] is that now the time-window, NT, is independent of the frequency-resolution of the BEM FIR, $1/(KT)$. This allows for an almost perfect fit of the BEM FIR to the true doubly-selective channel. Note that when NT is smaller than $1/(2f_{\max})$, a good fit can even be obtained with $Q = 2$.

To conclude, we have obtained a practical channel model for doubly-selective channels that is well-structured and contains a small number of parameters.

The BEM FIR input-output relation for $n \in \{0,1,\ldots,N-1\}$ can be written as

$$
y^{(m)}[n] = \sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(m)}e^{j2\pi qn/K}x[n-l] + w^{(m)}[n]. \qquad (6)
$$

3. SYSTEM MODEL

In this section, we try to rewrite (6) on a block level, which will turn out to be useful at a later stage. Defining the $(N+L)\times 1$ data symbol block $\mathbf{x} := [x[-L],\ldots,x[N-1]]^T$, the $N\times 1$ received sample block at the mth receive antenna $\mathbf{y}^{(m)} := [y^{(m)}[0],\ldots, y^{(m)}[N-1]]^T$ can be written as

$$
\mathbf{y}^{(m)} = \mathbf{H}^{(m)}\mathbf{x} + \mathbf{w}^{(m)}, \qquad (7)
$$

where $\mathbf{w}^{(m)}$ is similarly defined as $\mathbf{y}^{(m)}$, and $\mathbf{H}^{(m)}$ is the $N\times(N+L)$ matrix given by

$$
\mathbf{H}^{(m)} = \sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(m)}\mathbf{D}_q\mathbf{Z}_l, \qquad (8)
$$

where $\mathbf{D}_q := \text{diag}\{[1, e^{j2\pi q/K},\ldots,e^{j2\pi q(N-1)/K}]^T\}$ and $\mathbf{Z}_l := [\mathbf{0}_{N\times(L-l)}, \mathbf{I}_N, \mathbf{0}_{N\times l}]$. Substituting (8) in (7), we can write

$$
\mathbf{y}^{(m)} = \sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(m)}\mathbf{D}_q\mathbf{Z}_l\mathbf{x} + \mathbf{w}^{(m)}. \qquad (9)
$$

Defining $\mathbf{y} := [\mathbf{y}^{(0)T},\ldots,\mathbf{y}^{(M-1)T}]^T$, we finally obtain

$$
\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \qquad (10)
$$

where $\mathbf{w}$ is similarly defined as $\mathbf{y}$, and $\mathbf{H}$ is the $MN\times(N+L)$ matrix given by $\mathbf{H} := [\mathbf{H}^{(0)T},\ldots,\mathbf{H}^{(M-1)T}]^T$. Note that throughout this paper, we will assume perfect knowledge of $\mathbf{H}$. In practice, the BEM FIR coefficients have to be estimated. This can be done blindly [6, 10] or by training [12].

$$\boldsymbol{\mathcal{H}}^{(m)} := \begin{bmatrix} \boldsymbol{\Omega}^{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(m)} & \dots & \boldsymbol{\Omega}^{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(m)} & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \boldsymbol{\Omega}^{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(m)} & \dots & \boldsymbol{\Omega}^{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(m)} \end{bmatrix}$$

Based on (10), we can apply block equalization to recover $\mathbf{x}$ from $\mathbf{y}$. However, the complexity of such an approach depends on the block size N, which can often be very large. In this paper, we will therefore focus on serial equalization, for which the complexity is basically independent of the block size N. We focus on a non-precoded transmission, i.e., we assume that all entries of $\mathbf{x}$ contain raw data symbols. However, we will not estimate the edges of $\mathbf{x}$ and only estimate the middle part of $\mathbf{x}$ (denoted as $\mathbf{x}_\star$). The edges are either estimated in a previous step (top entries of $\mathbf{x}$) or will be estimated in a next step (bottom entries of $\mathbf{x}$). We will distinguish between serial linear equalization and serial decision feedback equalization.

4. SERIAL LINEAR EQUALIZATION

We adopt a Serial Linear Equalizer (SLE), consisting of a serial filter $f^{(m)}[n;\nu]$ for the mth receive antenna, in order to find an estimate of $x[n-d]$ (see Figure 1):

$$\hat{x}[n-d] = \sum_{m=0}^{M-1}\sum_{\nu=-\infty}^{\infty} f^{(m)}[n;\nu]y^{(m)}[n-\nu],$$

where d represents the synchronization delay. Since for the channel, the BEM FIR of (5) was applied, it is also convenient to use a BEM FIR for the serial filter $f^{(m)}[n;\nu]$. In other words, we design each serial filter $f^{(m)}[n;\nu]$ to have $L'+1$ TV taps, where the time-variation of each tap is modeled by $Q'+1$ complex exponentials:

$$f^{(m)}[n;\nu] = \sum_{l'=0}^{L'}\delta[\nu-l']\sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q' n/K} f_{q',l'}^{(m)}.$$

An estimate of $x[n-d]$ is then computed as

$$\hat{x}[n-d] = \sum_{m=0}^{M-1}\sum_{l'=0}^{L'}\sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q' n/K} f_{q',l'}^{(m)} y^{(m)}[n-l']. \quad (11)$$

4.1. Block Representation

Let us rewrite (11) on a block level. Defining the q'th frequency-shifted and l'th time-shifted received sequence related to the mth receive antenna as

$$\mathbf{y}_{q',l'}^{(m)} := \bar{\mathbf{D}}_{q'}\bar{\mathbf{Z}}_{l'}\mathbf{y}^{(m)},$$

where $\bar{\mathbf{D}}_{q'} := \text{diag}\{[1, e^{j2\pi q'/K}, \dots, e^{j2\pi q(N-L'-1)/K}]^T\}$ and $\bar{\mathbf{Z}}_{l'} := [\mathbf{0}_{(N-L')\times(L'-l')}, \mathbf{I}_{N-L'}, \mathbf{0}_{(N-L')\times l'}]$, and introducing

$$\mathbf{x}_\star := [x[L'-d], \dots, x[N-d-1]]^T,$$

an estimate of $\mathbf{x}_\star$ is obtained as

$$\hat{\mathbf{x}}_\star^T = \sum_{m=0}^{M-1}\mathbf{f}^{(m)T}\mathbf{Y}^{(m)}, \quad (12)$$

where $\mathbf{f}^{(m)}$ is the $(L'+1)(Q'+1)\times 1$ vector given by $\mathbf{f}^{(m)} := [f_{Q'/2,L'}^{(m)}, \dots, f_{Q'/2,0}^{(m)}, \dots, f_{-Q'/2,0}^{(m)}]^T$, and $\mathbf{Y}^{(m)}$ is the $(L'+1)(Q'+1)\times(N-L')$ matrix given by $\mathbf{Y}^{(m)} := [\mathbf{y}_{Q'/2,L'}^{(m)}, \dots, \mathbf{y}_{Q'/2,0}^{(m)}, \dots, \mathbf{y}_{-Q'/2,0}^{(m)}]^T$.

Let us now express $\mathbf{Y}^{(m)}$ as a function of the BEM FIR channel parameters and the data symbols. Using the property $\bar{\mathbf{Z}}_{l'}\mathbf{D}_q = e^{j2\pi q(L'-l')/K}\bar{\mathbf{D}}_q\bar{\mathbf{Z}}_{l'}$, the q'th frequency-shifted and l'th time-shifted received sequence related to the mth receive antenna can be written as

$$\begin{aligned}\mathbf{y}_{q',l'}^{(m)} &:= \bar{\mathbf{D}}_{q'}\bar{\mathbf{Z}}_{l'}\mathbf{y}^{(m)} \\ &= \sum_{l=0}^{L}\sum_{q=0}^{Q} h_{q,l}^{(m)} e^{j2\pi q(L'-l')/K}\bar{\mathbf{D}}_{q'}\bar{\mathbf{D}}_q\bar{\mathbf{Z}}_{l'}\mathbf{Z}_l\mathbf{x} + \mathbf{w}_{q',l'}^{(m)} \\ &= \sum_{l=0}^{L}\sum_{q=0}^{Q} e^{j2\pi q(L'-l')/K} h_{q,l}^{(m)}\bar{\mathbf{D}}_{q+q'}\tilde{\mathbf{Z}}_{l+l'}\mathbf{x} + \mathbf{w}_{q',l'}^{(m)},\end{aligned}$$

where $\mathbf{w}_{q',l'}^{(m)}$ is similarly defined as $\mathbf{y}_{q',l'}^{(m)}$ and $\tilde{\mathbf{Z}}_k := [\mathbf{0}_{(N-L')\times(L+L'-k)}, \mathbf{I}_{N-L'}, \mathbf{0}_{(N-L')\times k}]$. Introducing $k := l+l'$ and $p := q+q'$, and defining $\mathbf{x}_{p,k} := \bar{\mathbf{D}}_p\tilde{\mathbf{Z}}_k\mathbf{x}$ (note that $\mathbf{x}_\star = \mathbf{x}_{0,d}$), we can also write this as

$$\mathbf{y}_{q',l'}^{(m)} = \sum_{k=0}^{L+L'}\sum_{p=-(Q+Q')/2}^{(Q+Q')/2} e^{j2\pi(p-q')(L'-l')/K} h_{p-q',k-l'}^{(m)}\mathbf{x}_{p,k} + \mathbf{w}_{q',l'}^{(m)}.$$

Then, defining $\mathbf{X} := [\mathbf{x}_{Q/2+Q'/2,L+L'}, \dots, \mathbf{x}_{Q/2+Q'/2,0}, \dots, \mathbf{x}_{-Q/2-Q'/2,0}]^T$, $\mathbf{Y}^{(m)}$ can be expressed as

$$\mathbf{Y}^{(m)} = \boldsymbol{\mathcal{H}}^{(m)}\mathbf{X} + \mathbf{W}^{(m)},$$

where $\mathbf{W}^{(m)}$ is similarly defined as $\mathbf{Y}^{(m)}$ and $\boldsymbol{\mathcal{H}}^{(m)}$ is the $(Q'+1)(L'+1)\times(Q+Q'+1)(L+L'+1)$ matrix given at the top of this page, with $\boldsymbol{\mathcal{H}}_q^{(m)}$ the $(L'+1)\times(L+L'+1)$ Toeplitz matrix given by

$$\boldsymbol{\mathcal{H}}_q^{(m)} := \begin{bmatrix} h_{q,L}^{(m)} & \dots & h_{q,0}^{(m)} & & 0 \\ & \ddots & & \ddots & \\ 0 & & h_{q,L}^{(m)} & \dots & h_{q,0}^{(m)} \end{bmatrix},$$

and $\boldsymbol{\Omega} := \text{diag}\{[1, e^{j2\pi/K}, \dots, e^{j2\pi L'/K}]^T\}$. Defining $\mathbf{Y} := [\mathbf{Y}^{(0)T}, \dots, \mathbf{Y}^{(M-1)T}]^T$, we then obtain

$$\mathbf{Y} = \boldsymbol{\mathcal{H}}\mathbf{X} + \mathbf{W}, \quad (13)$$

where $\mathbf{W}$ is similarly defined as $\mathbf{Y}$ and $\boldsymbol{\mathcal{H}}$ is the $M(Q'+1)(L'+1)\times(Q+Q'+1)(L+L'+1)$ matrix given by $\boldsymbol{\mathcal{H}} := [\boldsymbol{\mathcal{H}}^{(0)T}, \dots, \boldsymbol{\mathcal{H}}^{(M-1)T}]^T$. Hence, (12) can be rewritten as

$$\hat{\mathbf{x}}_\star^T = \sum_{m=0}^{M-1}\mathbf{f}^{(m)T}\mathbf{Y}^{(m)} = \mathbf{f}^T\mathbf{Y} = \mathbf{f}^T\boldsymbol{\mathcal{H}}\mathbf{X} + \mathbf{f}^T\mathbf{W}, \quad (14)$$

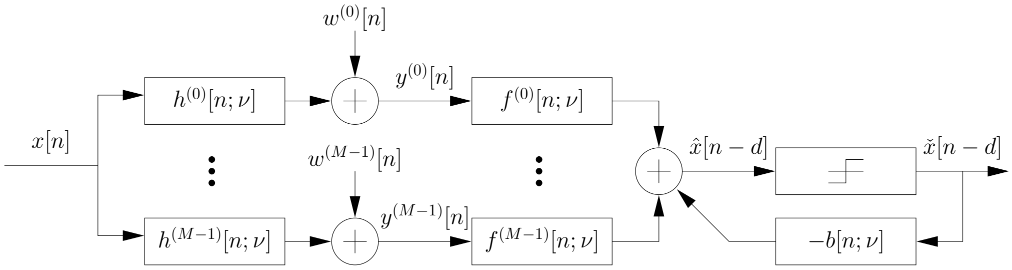

Fig. 2. Serial decision feedback equalization.

where $\mathbf{f}$ is the $M(L'+1)(Q'+1) \times 1$ vector given by $\mathbf{f} := [\mathbf{f}^{(0)T}, \ldots, \mathbf{f}^{(M-1)T}]^T$.

4.2. Equalizer Design

Let us focus on the Minimum Mean-Square Error (MMSE) SLE, which minimizes the MSE $\mathcal{J} = \mathrm{E}\{\|\mathbf{x}_\star - \hat{\mathbf{x}}_\star\|^2\}$. Defining the data and noise covariance matrices as $\mathbf{R}_X := \mathrm{E}\{\mathbf{X}\mathbf{X}^H\}$ and $\mathbf{R}_W = \mathrm{E}\{\mathbf{W}\mathbf{W}^H\}$, respectively, the MSE can be expressed as

$$\mathcal{J} = \mathbf{f}^T(\boldsymbol{\mathcal{H}}\mathbf{R}_X\boldsymbol{\mathcal{H}}^H + \mathbf{R}_W)\mathbf{f}^* - 2\Re\{\mathbf{e}^T\mathbf{R}_X\boldsymbol{\mathcal{H}}^H\mathbf{f}^*\} + \mathbf{e}^T\mathbf{R}_X\mathbf{e}^*,$$

where $\mathbf{e}$ is the $(Q+Q'+1)(L+L'+1) \times 1$ unit vector with a 1 in position $(Q+Q')(L+L'+1)/2 + d + 1$. Solving $\partial\mathcal{J}/\partial\mathbf{f} = \mathbf{0}$, we obtain

$$\begin{aligned}\mathbf{f}_{MMSE}^T &= \mathbf{e}^T\mathbf{R}_X\boldsymbol{\mathcal{H}}^H(\boldsymbol{\mathcal{H}}\mathbf{R}_X\boldsymbol{\mathcal{H}}^H + \mathbf{R}_W)^{-1} \\ &= \mathbf{e}^T(\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}\boldsymbol{\mathcal{H}} + \mathbf{R}_X^{-1})^{-1}\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}, \qquad (15)\end{aligned}$$

where the second equality is obtained by using the matrix inversion lemma.

Assuming the data sequence and the additive noises are mutually uncorrelated and white with variance σ_x^2 and σ_v^2, respectively, the data and noise covariance matrices are given by

$$\begin{aligned}\mathbf{R}_X &= \sigma_x^2\mathbf{J}_{Q+Q'+1} \otimes \mathbf{I}_{L+L'+1}, \\ \mathbf{R}_W &= \sigma_v^2\mathbf{I}_M \otimes \mathbf{J}_{Q'+1} \otimes \mathbf{I}_{L'+1},\end{aligned}$$

where $\mathbf{J}_I$ is the $I \times I$ matrix defined as

$$[\mathbf{J}_I]_{i,i'} = \sum_{n=0}^{N-L'-1} e^{j2\pi(i-i')n/K}.$$

5. SERIAL DECISION FEEDBACK EQUALIZATION

We adopt a Serial Decision Feedback Equalizer (SDFE), consisting of a serial feedforward filter $f^{(m)}[n;\nu]$ for the mth receive antenna and a serial feedback filter $b[n;\nu]$, in order to find an estimate of $x[n-d]$ (see Figure 2):

$$\begin{aligned}\hat{x}[n-d] = &\sum_{m=0}^{M-1}\sum_{\nu=-\infty}^{\infty} f^{(m)}[n;\nu]y^{(m)}[n-\nu] \\ &- \sum_{\nu=-\infty}^{\infty} b[n;\nu]\check{x}[n-d-\nu],\end{aligned}$$

where d again represents the synchronization delay and $\check{x}[n]$ is obtained after taking a decision on $\hat{x}[n]$. Since for the channel, the BEM FIR of (5) was applied, it is also convenient to use a BEM FIR for the serial feedforward filter $f^{(m)}[n;\nu]$ and the serial feedback filter $b[n;\nu]$. In other words, we design each serial feedforward filter $f^{(m)}[n;\nu]$ to have $L'+1$ TV taps, where the time-variation of each tap is modeled by $Q'+1$ complex exponentials:

$$f^{(m)}[n;\nu] = \sum_{l'=0}^{L'}\delta[\nu-l']\sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q'n/K} f_{q',l'}^{(m)},$$

and the serial feedback filter $b[n;\nu]$ to have $L''+1$ TV taps, where the time-variation of each tap is modeled by $Q''+1$ complex exponentials:

$$b[n;\nu] = \sum_{l''=0}^{L''}\delta[\nu-l'']\sum_{q''=-Q''/2}^{Q''/2} e^{j2\pi q''n/K} b_{q'',l''},$$

where in order to feedback decisions in a causal way, we require $b_{-Q''/2,0} = \cdots = b_{Q''/2,0} = 0$. An estimate of $x[n-d]$ is then computed as

$$\begin{aligned}\hat{x}[n-d] = &\sum_{m=0}^{M-1}\sum_{l'=0}^{L'}\sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q'n/K} f_{q',l'}^{(m)} y^{(m)}[n-l'] \\ &- \sum_{l''=1}^{L''}\sum_{q''=-Q''/2}^{Q''/2} e^{j2\pi q''n/K} b_{q'',l''}\check{x}[n-d-l''].\end{aligned} \qquad (16)$$

5.1. Block Representation

Let us rewrite (16) on a block level. Using the notation introduced in Section 4.1, and assuming that past decisions are correct, an estimate of $\mathbf{x}_\star$ is now obtained as

$$\begin{aligned}\hat{\mathbf{x}}_\star^T &= \sum_{m=0}^{M-1}\mathbf{f}^{(m)T}\mathbf{Y}^{(m)} - \mathbf{b}^T\mathbf{P}\mathbf{X} \\ &= \mathbf{f}^T\mathbf{Y} - \mathbf{b}^T\mathbf{P}\mathbf{X} = \mathbf{f}^T\boldsymbol{\mathcal{H}}\mathbf{X} - \mathbf{b}^T\mathbf{P}\mathbf{X} + \mathbf{f}^T\mathbf{W}, \qquad (17)\end{aligned}$$

where $\mathbf{b}$ is the $(L''+1)(Q''+1) \times 1$ vector given by $\mathbf{b} = [b_{Q''/2,L''}, \ldots, b_{Q''/2,1}, \ldots, b_{1,1}, b_{0,L''}, \ldots, b_{0,1}, 0, b_{-1,L''}, \ldots,$

$b_{-1,1}, \ldots, b_{-Q''/2,1}]^T$ and $\mathbf{P}$ is the $((Q''+1)L''+1) \times (Q+Q'+1)(L+L'+1)$ selection matrix given by

$$\mathbf{P} := \begin{bmatrix} & \mathbf{I}_{Q''/2} \otimes \mathbf{P}_1 & & & \\ \mathbf{0}_{\alpha \times \beta} & & \mathbf{P}_2 & & \mathbf{0}_{\alpha \times \beta} \\ & & & \mathbf{I}_{Q''/2} \otimes \mathbf{P}_1 & \end{bmatrix},$$

with $\alpha := (Q''+1)L''+1$, $\beta := (Q+Q'-Q'')(L+L'+1)/2$, $\mathbf{P}_1 := [\mathbf{0}_{L'' \times (L+L'-L''-d)}, \mathbf{I}_{L''}, \mathbf{0}_{L'' \times (d+1)}]$, and $\mathbf{P}_2 := [\mathbf{0}_{(L''+1) \times (L+L'-L''-d)}, \mathbf{I}_{L''+1}, \mathbf{0}_{(L''+1) \times d}]$.

5.2. Equalizer Design

Let us focus on the Minimum Mean-Square Error (MMSE) SDFE, which minimizes the MSE $\mathcal{J} = \mathrm{E}\{\|\mathbf{x}_\star - \hat{\mathbf{x}}_\star\|^2\}$. In a similar fashion as in Section 4.2, the MSE can be expressed as

$$\begin{aligned} \mathcal{J} = &\mathbf{f}^T(\boldsymbol{\mathcal{H}}\mathbf{R}_X\boldsymbol{\mathcal{H}}^H + \mathbf{R}_W)\mathbf{f}^* - 2\Re\{(\mathbf{b}+\mathbf{e})^T\mathbf{P}\mathbf{R}_X\boldsymbol{\mathcal{H}}^H\mathbf{f}^*\} \\ &+ (\mathbf{b}+\mathbf{e})^T\mathbf{P}\mathbf{R}_X\mathbf{P}^H(\mathbf{b}+\mathbf{e})^*, \end{aligned} \tag{18}$$

where $\mathbf{e}$ is the $((Q''+1)L''+1) \times 1$ unit vector with a 1 in position $Q''L''/2 + L'' + 1$. Solving $\partial\mathcal{J}/\partial\mathbf{f} = \mathbf{0}$, we obtain

$$\mathbf{f}_{MMSE}^T = (\mathbf{b}+\mathbf{e})^T\mathbf{P}\mathbf{R}_X\boldsymbol{\mathcal{H}}^H(\boldsymbol{\mathcal{H}}\mathbf{R}_X\boldsymbol{\mathcal{H}}^H + \mathbf{R}_W)^{-1} \tag{19}$$
$$= (\mathbf{b}+\mathbf{e})^T\mathbf{P}(\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}\boldsymbol{\mathcal{H}} + \mathbf{R}_X^{-1})^{-1}\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}, \tag{20}$$

where the second equality is again obtained by using the matrix inversion lemma. Next, substituting (19) in (18) results after some calculation into

$$\mathcal{J} = (\mathbf{b}+\mathbf{e})^T\mathbf{R}_{MMSE}(\mathbf{b}+\mathbf{e})^*,$$

where $\mathbf{R}_{MMSE} = \mathbf{P}(\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}\boldsymbol{\mathcal{H}}+\mathbf{R}_X^{-1})^{-1}\mathbf{P}^H$. Solving $\partial\mathcal{J}/\partial\mathbf{b} = \mathbf{0}$ under the constraint that $\mathbf{e}^T\mathbf{b} = 0$, we finally obtain

$$\mathbf{b}_{MMSE} = \frac{\mathbf{R}_{MMSE}^{-1}\mathbf{e}}{\mathbf{e}^T\mathbf{R}_{MMSE}^{-1}\mathbf{e}} - \mathbf{e}.$$

To summarize, we have

$$\begin{aligned} \mathbf{f}_{MMSE}^T &= \mathbf{b}_{MMSE}^T\mathbf{P}(\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}\boldsymbol{\mathcal{H}} + \mathbf{R}_X^{-1})^{-1}\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}, \\ \mathbf{R}_{MMSE} &= \mathbf{P}(\boldsymbol{\mathcal{H}}^H\mathbf{R}_W^{-1}\boldsymbol{\mathcal{H}} + \mathbf{R}_X^{-1})^{-1}\mathbf{P}^H, \\ \mathbf{b}_{MMSE} &= \frac{\mathbf{R}_{MMSE}^{-1}\mathbf{e}}{\mathbf{e}^T\mathbf{R}_{MMSE}^{-1}\mathbf{e}} - \mathbf{e}. \end{aligned}$$

6. SIMULATION RESULTS

In this section, we show a comparison between serial equalization and block equalization. Note that for block equalization we assume zero-padding based block transmission [15]. We generate M ($M = 1, 2$) channels consisting of 5 clusters with delays $-T$, $-T/2$, 0, $T/2$, and T. Assuming that $g_{\mathrm{tr}}(t)$ and $g_{\mathrm{rec}}(t)$ are rectangular functions over $[0, T)$ with height $1/T$, and thus $\psi(t) = g_{\mathrm{rec}}(t) \star g_{\mathrm{tr}}(t)$ is a triangular function over $[0, 2T)$ with height 1, we can thus assume that $L = 2$. The complex gain and frequency offset of the rth ray of the cth cluster are given by $G_{c,r}^{(0)} = e^{j\theta_{c,r}^{(0)}}/\sqrt{100}$ and $f_{c,r}^{(0)} = \cos(\phi_{c,r}^{(0)})f_{\max}$, where $\theta_{c,r}^{(0)}$ and $\phi_{c,r}^{(0)}$ are uniformly distributed over $[0, 2\pi)$. We use QPSK modulation. We assume the data sequence and the additive noises are mutually uncorrelated and white. The SNR is defined as $SNR = 5/\sigma_v^2$, where σ_v^2 is the variance of the additive noise. The factor 5

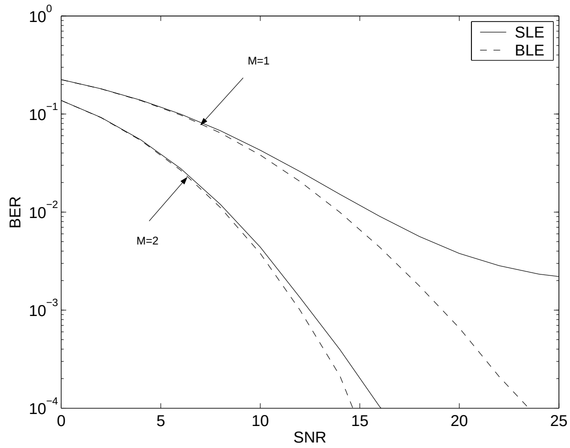

Fig. 3. Comparison of the SLE and the BLE in doubly-selective channels for $M = 1, 2$.

is due to the fact that we consider 5 clusters. We fit the BEM FIR to the true channel over a time window of $NT = 200T$, and use the obtained channel model parameters to design the equalizer. As already mentioned, when $NT \leq 1/(2f_{\max})$, which is the case here, an accurate channel model can be obtained by taking $Q = 2$. To satisfy $Q/(2KT) \approx f_{\max} = 1/(400T)$, we then take $K = 400$.

First, we compare the SLE with the Block Linear Equalizer (BLE). The BLE we adopt here is similar to the one developed for TIV channels (see [13, 7]). For the SLE, we consider $L' = 7$, $d = (L + L')/2 = 5$, and $Q' = 6$. Performance results for $M = 1, 2$ are plotted in Figure 3. We observe that the BLE and the SLE have a comparable performance, except for $M = 1$ at high SNR. This is due to the fact that for $M = 1$ there exsits a zero-forcing BLE whereas there exists no zero-forcing SLE.

Next, we compare the SDFE with the block decision feedback equalizer (BDFE). The BDFE we adopt here is similar to the one developed for TIV channels (see [14, 7]). For the SDFE, we consider $L' = 7$, $d = (L + L')/2 = 5$, $L'' = L + L' - d = 5$, $Q' = 6$, and $Q'' = (Q + Q')/2 = 4$. Performance results for $M = 1, 2$ are plotted in Figure 4. We observe that the BDFE and the SDFE have a comparable performance, for any M at any SNR.

Let us also focus on complexity for a moment. We can distinguish between design complexity and implementation complexity. The design complexity is the computational cost to design the equalizer, whereas the implementation complexity is the computational cost to equalize the channel once the equalizer has been designed. To compute the BLE (BDFE), we basically need $\mathcal{O}((N - L)^3)$ flops, where $(N - L)^3 \approx 7,645,400$ for this example. On the other hand, to compute the SLE (SDFE), we basically need $\mathcal{O}((Q+Q'+1)^3(L+L'+1)^3)$ flops, where $(Q+Q'+1)^3(L+L'+1)^3 \approx 970,300$ for this example. Hence, the design complexity of the SLE (SDFE) is clearly smaller than the design complexity of the BLE (BDFE). A similar observation holds for the implementation complexity, expressed in the number of Multiply-Add (MA) operations. The BLE requires $N(N - L) = 39,400$ MA operations per receive antenna, with an extra $(N - L)(N - L - 1)/2 = 19,306$ MA operations for the BDFE, whereas the SLE requires $(N - L')(Q' + 1)(L' + 1) = 10,808$ MA operations per receive antenna, with an extra $(N - L')(Q'' + 1)L'' = 4,825$ MA op-

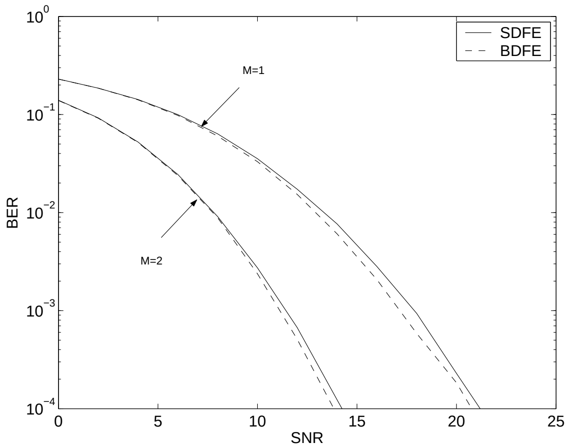

Fig. 4. Comparison of the SDFE and the BDFE in doubly-selective channels for $M = 1, 2$.

erations for the SDFE. One could argue that for a smaller block size N, the difference in complexity would disappear, but then the block size would not be large enough such that blind channel estimation performs reasonably or the overhead of the training symbols for training based channel estimation does not decrease the data transmission rate too much.

Note that in contrast to the results presented in [9, 2, 1], we do not see a BER performance saturation at high SNR due to a modeling error. The reason for this is that the modeling error of the BEM FIR has been significantly reduced by fitting the BEM FIR to the true doubly-selective channel over a time-window, NT, that is independent of the frequency-resolution of the BEM FIR, $1/(KT)$.

7. CONCLUSIONS

In this paper, we have proposed a novel serial equalization approach for doubly-selective channels, for which we have adopted a BEM FIR to model the doubly-selective channel and to design the serial equalizer. Both the SLE and the SDFE have been discussed. Simulation results have shown that the performance of the SLE is comparable with the performance of the BLE, except for a single receive antenna at high SNR. They have further shown that the performance of the SDFE is comparable with the performance of the BDFE, for any number of receive antennas at any SNR. On the other hand, the design and the implementation complexity of the SLE (SDFE) are generally smaller than the corresponding complexities of the BLE (BDFE).

8. REFERENCES

[1] I. Barhumi, G. Leus, and M. Moonen. Time-Varying FIR Decision Feedback Equalization of Doubly-Selective Channels. In *Proc. of IEEE Global Communications Conf. (GLOBECOM'03)*, San Francisco, California, December 2003.

[2] I. Barhumi, G. Leus, and M. Moonen. Time-Varying FIR Equalization of Doubly-Selective Channels. In *Proc. of IEEE Intl. Conf. on Communications (ICC'03)*, Anchorage, Alaska, May 2003.

[3] J. K. Cavers. *Mobile Channel Characteristics*. Kluwer, 2000.

[4] A. Duel-Hallen, S. Hu, and H. Hallen. Long-Range Prediction of Fading Channels. *IEEE Signal Processing Mag.*, pages 62–75, May 2000.

[5] G. B. Giannakis, Y. Hua, P. Stoica, and L. Tong, editors. *Signal Processing Advances in Wireless & Mobile Communications: Trends in Single- and Multi-User Systems*, volume 2. Prentice Hall, 2000.

[6] G. B. Giannakis and C. Tepedelenlioğlu. Basis Expansion Models and Diversity Techniques for Blind Equalization of Time-Varying Channels. *Proc. of the IEEE*, pages 1969–1986, 1998.

[7] A. Ginesi, G. M. Vitetta, and D. D. Falconer. Block Channel Equalization in the Presence of a Cochannel Interferent Signal. *IEEE Journal on Selected Areas in Communications*, 17(11):1853–1862, 1999.

[8] W. C. Jakes, editor. *Microwave Mobile Communications*. Wiley, 1974.

[9] G. Leus, I. Barhumi, and M. Moonen. MMSE Time-Varying FIR Equalization of Doubly-Selective Channels. In *Proc. of IEEE Intl. Conf. on Acoustics, Speech, and Signal Processing (ICASSP'03)*, Hong Kong, April 2003.

[10] G. Leus and M. Moonen. Deterministic Subspace Based Blind Channel Estimation for Doubly-Selective Channels. In *Proc. of the IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC'03)*, Rome, Italy, June 2003.

[11] H. Liu and G. B. Giannakis. Deterministic Approaches for Blind Equalization of Time-Varying Channels with Antenna Arrays. *IEEE Trans. on Signal Processing*, 46(11):3003–3013, November 1998.

[12] X. Ma, G. B. Giannakis, and S. Ohno. Optimal Training for Block Transmissions over Doubly-Selective Fading Channels. *IEEE Trans. on Signal Processing*, 2003. to appear; See also Proc. ICASSP, May 2002.

[13] A. Scaglione and G. B. Giannakis. Redundant Filterbank Precoders and Equalizers, Part I: Unification and Optimal Designs. *IEEE Trans. on Signal Processing*, 47(7):1988–2006, July 1999.

[14] A. Stamoulis, G. B. Giannakis, and A. Scaglione. Block fir decision-feedback equalizers for filterbank precoded transmissions with blind channel estimation capabilities. *IEEE Trans. on Communications*, 49(1):69–83, January 2001.

[15] Z. Wang and G. B. Giannakis. Linearly Precoded or Coded OFDM against Wireless Channel Fades? In *Proc. of 3rd IEEE Workshop on Signal Processing Advances in Wireless Communications*, pages 267–270, Taoyuan, Taiwan, R.O.C., April 2001. Submitted also to *IEEE Transactions on Information Theory*, April 2001.